

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequalities [2022]

Level - 2	Daily Tutorial Sheet 10
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116.(C) Since, $a(x^2 + 4x + 4)^2 + b(x^2 + 4x + 4) + c = 0 \Rightarrow a(x+2)^4 + b(x+2)^2 + c = 0$

Put $(x+2)^2 = t \Rightarrow at^2 + bt + c = 0$

Now, $\Delta = b^2 - 4ac \Rightarrow \Delta > 0 \quad \{\because b^2 > 4ac \text{ (Given)}\}$

$\Rightarrow$ The roots of the equation are real and distinct. Let the roots be α and β .

$\therefore (x+2)^2 = \alpha \text{ and } (x+2)^2 = \beta \Rightarrow \alpha \text{ and } \beta \text{ are both +ve} \Rightarrow \alpha + \beta > 0 \text{ and } \alpha\beta > 0$

$\Rightarrow -\frac{b}{a} > 0 \text{ and } \frac{c}{a} > 0 \text{ i.e. } a \text{ and } c \text{ are of same sign and } b \text{ is of opposite sign.}$

117.(B) $(\lambda + 2)(\lambda - 1)x^2 + (\lambda + 2)x - 1 < 0 \quad \forall x \in R$

$\Rightarrow (\lambda + 2)(\lambda - 1) < 0 \Rightarrow -2 < \lambda < 1 \quad \dots\dots\dots(i) \quad (a < 0)$

$\Rightarrow (\lambda + 2)(\lambda + 2 + 4\lambda - 4) < 0 \quad (D < 0) \Rightarrow (\lambda + 2)(5\lambda - 2) < 0$

$\Rightarrow -2 < \lambda < \frac{2}{5} \quad \dots\dots\dots(ii)$

(i) and (ii) $\Rightarrow \lambda \in \left(-2, \frac{2}{5}\right)$

Also $\lambda = -2 \Rightarrow 0 < 1$ which is true $\therefore$ Required interval is $\lambda \in \left[-2, \frac{2}{5}\right)$

118.(C) Given equation $(k - 2)x^2 + 8x + k + 4 = 0 \quad \dots\dots\dots(i)$

Since roots are real and distinct, therefore,

$8^2 - 4(k - 2)(k + 4) > 0 \Rightarrow 64 - 4\{k^2 + 2k - 8\} > 0$

$\Rightarrow k^2 + 2k - 24 < 0 \Rightarrow (k + 6)(k - 4) < 0$

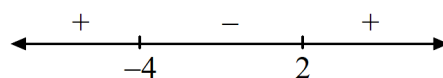
$\Rightarrow -6 < k < 4 \quad \dots\dots\dots(ii)$

Also, both the roots are negative, therefore, 'S' < 0 and 'P' > 0

$S < 0 \Rightarrow \frac{-8}{k - 2} < 0 \Rightarrow \frac{1}{k - 2} > 0 \Rightarrow k - 2 > 0$

$\Rightarrow k > 2 \quad \dots\dots\dots(iii)$

$\Rightarrow P > 0 \Rightarrow \frac{k + 4}{k - 2} > 0 \Rightarrow k \in (-\infty, -4) \cup (2, \infty) \quad \dots\dots\dots(iv)$



From (ii), (iii) and (iv) $2 < k < 4 \Rightarrow k = 3$

119.(B) $mx^2 - 9mx + 5m + 1 = m(x^2 - 9x + \frac{81}{4}) + (5m - \frac{81m}{4} + 1) = m\left(x - \frac{9}{2}\right)^2 + \left(1 - \frac{61m}{4}\right)$

And this expression is positive for all $x \in R$ if $m \geq 0$ and $1 - \frac{61m}{4} \geq 0$,

(When $m = 0$, given expression = 1)

i.e. if $m \geq 0$ and $\frac{61m}{4} \leq 1$ i.e. if $0 \leq m \leq \frac{4}{61}$

120.(C) Let α be the common root.

Then $\alpha, 4\beta$ are the roots of $x^2 - 6x + \alpha = 0$ and $\alpha, 3\beta$ are the roots of $x^2 - cx + 6 = 0$

$$\alpha + 4\beta = 6 \Rightarrow \alpha + \frac{8}{\alpha} = 6 \quad [\text{using } 3\alpha\beta = 6] \Rightarrow \alpha^2 - 6\alpha + 8 = 0 \Rightarrow (\alpha - 2)(\alpha - 4) = 0 \quad \alpha = 2 \text{ or } 4$$

If $\alpha = 2$ then $\beta = 1 \in \mathbb{Z}$; if $\alpha = 4$ then $\beta = \frac{1}{2} \notin \mathbb{Z} \therefore$ the required value of α is 2.

121.(C) The roots are equal and opposite in sign so, sum of the roots $= 0 \Rightarrow -\frac{b}{a} = 0 \Rightarrow b = 0$

The roots are of opposite sign $\Rightarrow \frac{c}{a} < 0 \Rightarrow a, c$ have opposite signs.

122.(B) If $\alpha^2 = 5\alpha - 3, \beta^2 = 5\beta - 3 \Rightarrow \alpha, \beta$ are roots of the equation $x^2 - 5x + 3 = 0 \Rightarrow \alpha + \beta = 5, \alpha\beta = 3 \dots (i)$

Let the equation be $x^2 - Sx + P = 0$ whose roots are α/β and β/α

$$\text{Sum of roots: } \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{25 - 6}{3} = \frac{19}{3} \quad \text{and} \quad \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$$

Required equation is $3x^2 - 19x + 3 = 0$

123.(A) We can rewrite the given equation $ax^2 - a(b+c)x + abc + x - d = 0$

$$\Rightarrow a(x-b)(x-c) + x - d = 0$$

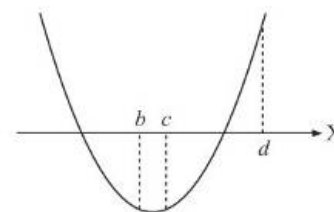
$$\text{Let } f(x) = a(x-b)(x-c) + x - d$$

As $a > 0, y = f(x)$ represents a parabola which opens upwards.

$$\text{And also, } f(b) = b - d < 0, f(c) = c - d < 0$$

$$\text{And } f(d) = a(d-b)(d-c) > 0$$

Thus, $f(x) = 0$ has exactly one root lying in $(-\infty, b)$ and one root in (c, d)



124.(C) Sum of roots $|\alpha| + |\beta| = -\alpha - \beta \quad (\because \alpha < 0 \text{ and } \beta < 0)$

$$= -(\alpha + \beta) = -\left(-\frac{b}{a}\right) = \frac{b}{a} = \left|\frac{b}{a}\right| \quad (\because |\alpha| + |\beta| > 0)$$

And product of roots $= |\alpha||\beta| = |\alpha\beta| = \left|\frac{c}{a}\right|$. Hence, required equation is $x^2 - \left(\left|\frac{b}{a}\right|\right)x + \left|\frac{c}{a}\right| = 0$

$$\text{or } |a|x^2 - |b|x + |c| = 0$$

125.(B) The roots of $bx^2 + cx + a = 0$ are imaginary $\Rightarrow c^2 - 4ab < 0 \Rightarrow c^2 < 4ab$

Again, the coefficient of x^2 in $3b^2x^2 + 6bcx + 2c^2$ is +ve, so the minimum value of the expression

$$= \frac{-\left(36b^2c^2 - 4(3b^2)(2c)^2\right)}{4(3b^2)} = -\frac{12b^2c^2}{12b^2} = -c^2$$

As $c^2 < 4ab$ we have $-c^2 > -4ab$. Hence $3b^2x^2 + 6bcx + 2c^2 > -4ab \forall x \in R$